AN ANALYSIS OF CO-MOVEMENT IN THE TURKISH REAL SECTOR CREDIT DEFAULT PROBABILITY AND EARLY WARNING INDICATORS: A CASE OF TURKISH REAL SECTOR

Evren Bolgün **Barış Akçay** **Tunç Oygur**

Özet

Türkiye'nin 2001 finansal krizi sonrasında bankacılık sektöründen başlayarak almış olduğu yapısal önlemler, reel sektöre yönelik kredi imkanlarını uzun bir süre olumlu yönde etkilemiştir. Ancak 2008 global konut krizi sonrasında değişen finansal borçlanma imkanları ile Türk Reel sektörünün hızla yükselen döviz yükümlülükleri yerel finansal kırılganlıkları arttırmıştır. Bu ilişkinin ortaya konması için reel sektörün temerrüt olasılıklarının tahmin edilmesi gerekmektedir. Belirsizliğin yönetilmesi için, kredi risk göstergeleri ile sektörel temerrüt tahminleri arasındaki dinamik bağlantının belirlenmesi önem taşımaktadır.

Araştırma kapsamında; ilk olarak, 2001-2017 döneminde BIST-100 içerisindeki bankacılık sektörü hariç tüm reel sektör firmalarının üç aylık konsolide bilanço verileri ve borsa piyasa değerleri dikkate alınarak firma temerrüt olasılıkları hesaplanmıştır. İkinci aşamada ise, elde edilen tahmin verileri ile reel sektör firmalarının sektörel bazda gerçekleşen temerrüt verileri arasındaki dinamik ilişkiler incelenmiştir. Daha sonra bu verinin kredi riski öncü göstergesi olarak kabul edilen Reel/Bankacılık sektörleri ve Türkiye CDS değerlerini arasındaki ilişkileri analiz edilmiştir.

Temerrüt olasılıklarının hesaplanmasında Merton (1974) tarafından geliştirilen opsiyon fiyatlama modeli temelli değerleme metodolojisi baz alınmıştır. Firma temerrüt olasılıkları ile CDS fiyatlamaları arasındaki doğrusal eşbütünleşme (cointegration) ve doğrusal olmayan dinamik dalgacık analizi (wavelet coherence) yaklaşımları ile ilişki düzeyleri analiz edilmiştir.

Anahtar kelimeler: *Kredi Temerrüt Olasılığı, CDS, Eşbütünleşme, Dalgacık Analizi*

Abstract

After the 2001 Turkish Financial Crises, the structural precautions taken in the Banking Sector gives positive incentives to the Turkish Real Sector as well by the credit channels. In fact, the 2008 Global Mortgage Crises changes the local financial borrowing conditions and demand of high concentrations in the Turkish Real Sector FX Liability increases local financial fragility conditions. In order to understand the uncertainty & relation of FX borrowing vulnerability in the Real Sector of Turkey it is important to define the dynamic credit risk indicators and sectoral default predictions.

In the research, we used the 2001-2007 period data's of the BIST-100 firms (excluding banks) by 3 month Consolidated Balance Sheet Financials, Firm Stock Exchange Market Values, CBRT Real Sector Credit Default History, Turkish Bank's NPL Records, Real Sector Firm CDS Historical Prices in order to explore the credit default probabilities. On the second phase, we analyzed the dynamic relations between the calculated prediction data's & sectoral based ex-post Real Sector Firms default results. Furthermore as it is accepted a credit risk leading indicator, we analyzed the relation between the Real/Banking sector data & Turkish Sovereign CDS values.

We used the well-known Merton (1974) model based on the option valuation methodology principles. We also searched the co-integration & dynamic nonlinear wavelet coherency analysis between the firm credit default probability & CDS price changes in the same period.

Keywords: *Credit Default Probability, CDS, Cointegration, Wavelet Coherence*

1. Introduction

Credit risk is well known as the potential that a borrower will fail to meet its obligations in accordance with the agreed terms. Generally in terms of individual & institutional investors perspective, bonds and other tradable debt instruments are the main source of credit risk. In fact, for banks, loans are the largest and most obvious source of credit risk. Banks need to manage their credit risk exposure in the whole credit portfolio as well as the risk in their individual credits or their financial transactions. After 2008 global financial crises most of the world's largest financial institutions have developed advanced risk management systems in order to model their exposure to credit risk. Such models are intended to help institutions in quantifying, monitoring and managing risk across their business lines. Default credit models represent a strategic decision of the set of quantitative tools required by financial institutions. A well known default credit risk model for public firms performs a critical role by helping financial analysts to make informed credit decisions by associating default probability with borrower firms and counterparties. Such a model can be used as a monitoring tool for screening obligors, for performing risk/return analysis of credit portfolios or for capital allocation and loan pricing.

However, the analysis of historical financial statements may present an incomplete or distorted picture of the company's true financial condition. For a variety of reasons including the intrinsic conservatism of accounting principles, financial statements do not necessarily reflect the complete economic reality of the firm. Furthermore, accounting practices do not provide a means for expressing uncertainty about the future since the fundamental principle is to "account" for all the items involved in the firm's operations during every period precisely. Unfortunately, while financial statements provide information directly about a firm's past, they are limiting in that they provide information only indirectly about its future.

In particular, the Merton model relies heavily on economic theories about market efficiency. The model contains embedded assumptions about the comprehensiveness of the information contained in market data when used within the structure of the model. However, knowledge of the market information alone does not directly inform an investor as to a borrower's creditworthiness.(1)

Measuring a firm's probability of default (PD) is one of the central problems of credit risk analysis. Moody's Analytics' Public Firm EDF (Expected Default Frequency) model has been the industry-leading PD model since its introduction in the early 1990s. Since that time, the model has undergone considerable development, and it continues to evolve, while providing unequalled global coverage of public firms on a daily basis.

While many readers are comfortable with the underlying theoretical framework of the KMV structural model, others may be new to it. Many authors provide useful descriptions of how the approach works using option-pricing theory. For in-depth overviews, please refer to Crosbie and Bohn (2003), Ranson (2005), Caouette, et.al (2008), Bohn and Stein (2009), Duffie and Singleton (2012).

The Public Firm EDF model belongs to the class of structural credit risk models, pioneered by Fischer Black, Robert Merton, and Myron Scholes. This model, originated by KMV takes an option-pricing based approach to credit risk. Only when an option contract expires in-the-money, does it receive a payoff. Naturally, the valuation reflects the probability of the contract expiring out-of-the-money. Due to limited liability laws, the market value of a publicly traded company's equity is lower bounded at zero, giving it call option-like characteristics. Hence, the default probability is embedded in the stock price. The stock price, however, contains other information as well. This complexity makes extracting the PD an interesting problem. Option pricing research provides a mathematical structure, within which, we can study default probabilities. (2)

(1) Moody's Investor Service.(March 2000), Rating Methodolgy, Moody's Public Firm Risk Model: A Hybrid Approach To Modeling Short Term Default Risk

(2) Moody's Analytics, Modelling Methodolgy, Credit Risk Modeling of Public Firms,pg.3-4

The base case is a firm that has a single class of debt and equity, which is the simplest nontrivial capital structure. In some way or another, according to the contractual arrangements for each, they share in whatever happens to the assets, good or bad. Recognizing that, what we did was to consider the assets as having a certain market value. For publicly traded companies, one could get that market value by adding up the market prices of all the liabilities plus equity, which has to equal, definitionally, the market value of the assets. If you plot the payoff to equity at the maturity of the debt, say in five years, against the value of the firm's assets, you would see that the payoff structure looks identical to the structure for a call option, except the call option is not on just the stock. The call option is on the whole firm, or the market value of the assets of the firm. From these terms, you recognize that you can value the leveraged equity of the firm as if it were a call option on the assets of the firm. If we have a way to value equities as an option, then we can value the debt by subtraction. We take the total market value of assets, subtract from it the value of the option (the option-type structure that equity represents), and end up with the value of the debt. So, that's how you arrive at the valuation of the debt. Once you have a value function for the debt and a value function for the equity using an option-pricing-type structure, then you can also figure out the risk of the debt and all the Greeks of traditional option pricing. You can say: What's the delta (that is, what is the sensitivity of debt value and equity value to a change in asset value) or the sensitivity of debt value and equity value to a change in the risk-free interest rate, asset value volatility, and so forth? And what is the effect of the changes in the volatility of the value of assets? So, it's an analogous structure. It was really recognizing Modigliani and Miller's observation that the right side of the balance sheet, liabilities plus equity, is always equal to the total assets on the left side of the balance sheet and then recognizing that the payoff structure to equity was just like an option. (3)

In this paper, we used the 2001-2007 period data's of the BIST-100 firms (excluding banks) by 3 month Consolidated Balance Sheet Financials, Firm Stock Exchange Market Values, CBRT Real Sector Credit Default History, Turkish Bank's NPL Records, Real Sector Firm CDS Historical Prices in order to explore the credit default probabilities. On the second phase, we analyzed the dynamic relations between the calculated prediction data's & sectoral based ex-post Real Sector Firms default results. Furthermore as it is accepted a credit risk leading indicator, we analyzed the relation between the Real/Banking sector data & Turkish Sovereign CDS values. We used the well-known Merton (1974) model based on the option valuation methodology principles. We also searched the co-integration & dynamic nonlinear wavelet coherency analysis between the firm credit default probability & CDS price changes in the same period.

2. Literature Review

Over the past several years, number of researchers have examined the contribution of the Merton model. The first authors to examine the model carefully were practitioners employed by either KMV or Moody's. Crosbie and Bohn (2003) summarize KMV's default probability model. Several papers addressing the accuracy of the KMV Merton model are available on the internet. Stein (2002) argues that KMV-Merton models can easily be improved upon. Other papers, including Bohn, Arora and Korablev (2005), argued that KMV-Merton models capture all of the information in traditional agency ratings and well known accounting variables. Both Hillegeist, Keating, Cram and Lundstedt (2004) and Du and Suo (2004) examine the model's predictive power. Duffie and Wang (2004) show that KMV-Merton probabilities have significant predictive power in a model of default probabilities over time, which can generate a term structure of default probabilities. Farmen, Westgaard et al (2003) investigate the default probabilities and their comparative statics (default Greeks) in the Merton framework using the objective or 'real' probability measure.

(3) A Model in Mind, Robert C.Merton, CFA Magazine July-August 2004

Bohn (2000) surveys some of the main theoretical models of risky debt valuation that built on Merton (1974) and Black and Cox (1976). Empirical evidence has suggested that the actual credit spreads are higher than model spreads. Jones, Mason, and Rosenfeld (1983) and Frank and Torous (1989) find that contingent-claim models yield theoretical credit spreads much lower than actual credit spreads. In the same year, Sarig and Warga (1989) estimate the term structure of credit spreads and show it to be consistent with contingent claim model predictions. A more recent study by Wei and Guo (1997) tests the models of Merton (1974) and Longstaff and Schwartz (1995) and finds the Merton model to be empirically superior. However, Gemmill (2002) employs zero coupon corporate bonds data and concludes that model and market spreads are on average of similar magnitude. Similar to previous research, market spreads are high (relative to model spreads) for bonds which have low risk and for bonds which are near to maturity. Longstaff and Schwartz (1995) argue that an additional element in the spread is the expectation that equity holders and other junior claimants receive in the bankruptcy settlement more than what is consistent with absolute Priority rule. In addition, Anderson and Sundaresan (1996) suggest that debt holders are forced to accept concessions to receive less than the originally agreed amount, prior to formal bankruptcy proceedings.

Mella-Barral and Perraudin (1997) incorporate this strategic debt service into an option pricing-based model and show that the spread widening impact can be significant. The upshot of the study is that the simple structural models (eg. Merton,Geske) forecast spreads which are smaller than market spreads, particularly for companies which have low leverage and low volatility, but the more complicated structural models which produce larger spreads (eg. Longstaff/Schwartz and Leland/Toft) also produce large errors. Another finding is that whether a model allows for stochastic interest rates or not does not make much difference. (4)

Clearly, default risk is of great interest not only to bond holders, but to owners of equity as well. They are strongly infuenced by bond defaults. However, though simple to state, it is not immediately obvious either how to measure default risk or how to model it. On the one hand, the causes of default risk, from loss of competitiveness, to a weak economy, to misperceptions of risk and return, to corporate mismanagement, are many and often hidden within the company. As outlined in Crouhy, Galai, and Mark (2000) credit risk may also become manifest in a multitude of ways. From downgrades, actual defaults, and other company specific factors to changes in market indices, general economic factors, and interest, exchange, and unemployment rates, both the causes and the manifestations of changes in credit conditions are complex. Nonetheless, ultimately, the issue of default risk boils down to the question of: "Is there sufficient asset value in the company to pay the obligations due?"

The problem of how to measure and manage default risk, in particular that associated with corporations is as old as the concept of the company itself. Prior to the 1950s, most techniques focused on traditional accounting and financial statement-analysis methods. Franco Modigliani was the first to place the problem within the theoretical context now recognizable as modern finance.

Along with coauthor Merton Miller, Franco Modigliani (1958) rigorously proposed scaffolding for the exploration of the relationship between a company's market value and its debt and equity .nancing. An explicit equivalency linking the value of a company to its financial structure, expressed in terms of bonds, equity, and derivative securities based on these was established.

The 1960s and 1970s saw an explosive growth in the use of equity options culminating with the founding of the Chicago Board Options Exchange, CBOE, in 1973. The ready existence of a liquid market for derivative securities allowed for new types of analysis. Black and Scholes (1973) realized that what market makers actually do is to take risk-neutral positions in the contracts they deal with and make their money off the bid-ask spread.

(4) How Good Is Merton Model At Assessing Credit Risk? Evidence From India, Amit Kulkarni, Alok Kumar Mishra, Jigisha Thakker,pg.17-18

Therefore, the price of an option is determined by the costs involved in creating a risk-neutral portfolio. Under this paradigm, it becomes clear that it is stock-price volatility that determines the prices for both puts and calls. In fact, for this reason, traders are just as likely to quote volatility as they are to quote price.

Merton (1974), one year later, utilized this same methodology, treating the value of corporate debt, from the perspective of derivative pricing, in order to study the risk structure of corporate bonds. The Modigliani-Miller (1958) and Merton (1974) results follow from the proposition that the capital structure does not a¤ect the company.s asset value. Although, as shown in recent papers, applying Black-Scholes and option pricing, the stock price of the company can be impacted by the capital structure, yet the asset value, which is split up into stocks and bonds, is independent of the capital structure.9 Those results are, however, obtained by assuming an exogenous stochastic process, a Brownian motion, for the asset value, which does not originate, as we will argue later, from the solution of a dynamic decision problem of a company acting under constraints. In other words, because of the complexity of the underlying company's value-debt dynamics, it is tempting to build models that do not depend upon them, i.e., to make no attempt to offer a causal explanation for the phenomena.

Credit spread models, for example, treat the problem by considering the spread between the interest rate on defaultable debt and that of similar maturity risk-free debt. The idea here is that the reason for the spread is that bond purchasers need to be compensated for the risk present in the former and that this will yield information about the probability of default. Jonkhurt's (1979) paper is one of the first to discuss the credit spread approach, while Hull and White (2000) have a more recent treatment. Another popular approach is the intensity model. Whereas the company-value method attempts to link default frequency to fundamental processes related to the .nancial structure of a company, an intensity model only seeks to describe the statistical characteristics of these events. Thus, like the credit spread approach, it offers little explanation of the fundamental default process. Madan and Unal (1998) use intensity-based methods in their paper and Du¢ e and Singleton (1997) develop the topic within the context of factor models.

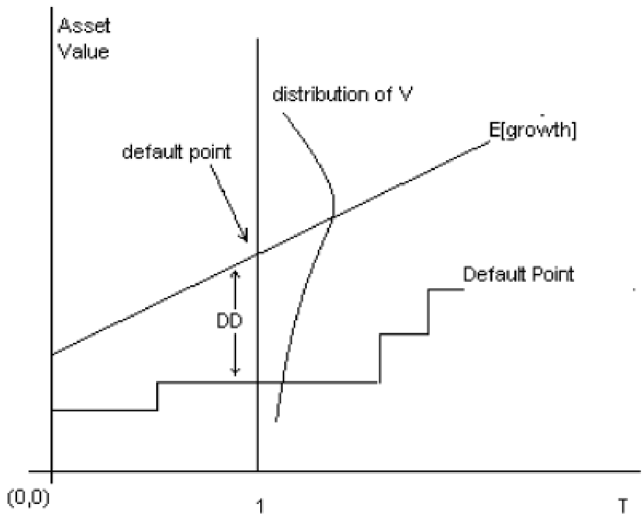
The rise of structured financial products, e.g., CDOs, wherein collections of risky products are grouped together, has greatly increased the interest in default correlation models. Through the use of copula functions and other methods, it is possible to relate the default dependency internal to complex products to a generalized correlation variable. This framework also allows for the discussion of correlated defaults within the context of both intensity and company value models. Douglas Lucas.(1995) paper is one of the first to explicitly discuss the topic, whereas Schönbucher (2001) and Embrechts, Lindskog, and McNeil (2003) present more contemporary treatments. Das and Du¢ e (2005) present evidence on how default events de.nitely correlate to a greater degree than had been thought. In contrast, our study is interested in correlations between "input" variables, i.e., stochastic shocks to different elements of a company's capital assets and how those shocks ultimately influence the probability of default.

Moody's KMV (named for Kealhover, McQuown, and Vasicek, cofounders of the KMV Corporation) model calculates the Expected Default Frequency (EDF) based on the company's capital structure, the volatility of the assets returns and the current asset value. The model specifies the financial structure of the company in terms of assets, current debt, long-term debt, and preferred shares. Next, the default point (DPT), the asset value where the company defaults, is computed. It is assumed that this point is above the size of its short-term debt.

The distance-to-default, *DD,* is the number of standard deviations between the mean of the distribution of the assets value and the default point, where $E[Vgrowth]$ = *Expected* $[Asset\ Value\ in\ 1\ Year]$*, Default Point = (short term debt) + ½ (long term debt), and σ = (volatility of asset returns).*

The last stage in this procedure is to construct a large list of companies, calculate their respective *DDs*, and note the expected default frequency, *EDF*, as a function of *DD*. Thus an estimate of the *EDF*; based on valuation, capital structure, and the market as a whole is achieved. Thus, this model combines structural elements and historical data to estimate probability of default.(5)

Figure 1. Distance to Default Model



3. Methodology

3.1. Analysis of Default Probability

Probability at default is calculated in the following prosecudure:

(1) $E_0 = call\ option\ premium = A_0 \times N(d_1) - D \times e^{-rT} \times N(d_2)$

$$d1 = \frac{\ln\left(\frac{A_0}{D}\right) + (r + 0.5{\sigma_A}^2) \times T}{\sigma_A\sqrt{T}};\ d2 = d1 - \sigma_A\sqrt{T}$$

(2) $\sigma_E = \frac{A_0}{E_0} \times N(d1) \times \sigma_A$

Where E is a total market value, σ_E is stock return volatility, A is a market value of firm's asset and σ_A the volatility of the firm's asset and D indicates total dept. Equation (1) and (2) are solved together and found the volatility of firm's asset.

(3) $DP = Short\ term\ debt + \frac{1}{2} Long\ term\ debt$

(4) $DD = \frac{A_T - DP}{\sigma_A * A_T}$

(5) $PD = N(-DD)$

Where DP is default point and DD denotes distance-to-default and PD is probability at default.

3.2. Wavelet Analysis

3.2.1. The continuous wavelet transform (CWT)[1]

A wavelet is a function with zero mean and that is localized in both frequency and time. We can characterize a wavelet by how localized it is in time (Δt) and frequency ($\Delta\omega$ or the bandwidth).

[1] The description of CWT, XWT and WTC is heavily drawn from Grinsted et al. (2004). We are grateful to Grinsted and co-authors for making codes available at: http://www.pol.ac.uk/home/research/waveletcoherence which was utilized in the present study.

The classical version of the Heisenberg uncertainty principle tells us that there is always a tradeoff between localization in time and frequency. Without properly defining Δt and $\Delta\omega$, we will note that there is a limit to how small the uncertainty product $\Delta t \cdot \Delta\omega$ can be. One particular wavelet, the Morlet, is defined as

$$\psi_0(\eta) = \pi^{-1/4} e^{i\omega_0\eta} e^{-\frac{1}{2}\eta^2}.$$

where ω_0 is dimensionless frequency and η is dimensionless time. When using wavelets for feature extraction purposes the Morlet wavelet (with $\omega_0 = 6$) is a good choice, since it provides a good balance between time and frequency localization. We therefore restrict our further treatment to this wavelet.

The idea behind the CWT is to apply the wavelet as a bandpass filter to the time series. The wavelet is stretched in time by varying its scale (s), so that $\eta = s \cdot t$ and normalizing it to have unit energy. For the Morlet wavelet (with $\omega_0 = 6$ the Fourier period (λ_{wt}) is almost equal to the scale ($\lambda_{wt} = 1.03s$). The CWT of a time series ($x_n, n = 1, \ldots, N$) with uniform time steps δt, is defined as the convolution of x_n with the scaled and normalized wavelet. We write

$$W_n^X(s) = \sqrt{\frac{\delta t}{s}} \sum_{n'=1}^{N} x_{n'} \psi_0[(n' - n)\frac{\delta t}{s}].$$

Although it is possible to calculate the wavelet transform using the above formula for each value of s and n, one can also identify the computation for all the values of n simultaneously as a convolution of two sequences. The standard procedure is to calculate this convolution as a simple product in the Fourier domain, using the Fast Fourier Transform algorithm to go forth and back from time to spectral domain. As with other types of transforms, the CWT applied to a finite length time-series inevitably suffers from border distortions, which increase with s. The region in which the transform suffers from these edge effects is called the Cone of Influence (COI). In this area, the results are unreliable and have to be interpreted carefully. Here we take the COI as the area in which the wavelet power caused by a discontinuity at the edge has dropped to e^{-2} of the value at the edge.

We define the wavelet power as $|W_n^X(s)|^2$. The complex argument of $W_n^X(s)$ can be interpreted as the local phase. The statistical significance of wavelet power can be assessed relative to the null hypotheses that the signal is generated by a stationary process with a given background power spectrum (P_k). Although Torrence and Compo (1998) have shown how the statistical significance of wavelet power can be assessed against the null hypothesis that the data generating process is given by an AR(0) or AR(1) stationary process with a certain background power spectrum (P_k), for more general processes one has to rely on Monte-Carlo simulations. Torrence and Compo (1998) computed the white noise and red noise wavelet power spectra, from which they derived, under the null, the corresponding distribution for the local wavelet power spectrum at each time n and scale s as follows:

$$D(\frac{|W_n^X(s)|^2}{\sigma_X^2} < p) = \frac{1}{2} P_k \chi_v^2(p),$$

where v is equal to 1 for real and 2 for complex wavelets.

3.2.2.The cross wavelet transform (XTC)

The cross wavelet transform (XWT) of two time series x_n and y_n is defined as $W^{XY} = W^X \cdot W^{Y*}$, where W^X and W^Y are the wavelet transforms of x and y, respectively, $*$ denotes complex conjugation.We further define the cross wavelet power as $|W^{XY}|$. The complex argument $\arg(W^{xy})$ can be interpreted as the local relative phase between x_n and y_n in time frequency space. The theoretical distribution of the cross wavelet power of two time series with background power spectra P_k^X and P_k^Y is given in Torrence and Compo (1998) as

$$D\left(\frac{|W_n^X(s)W_n^{Y*}(s)|}{\sigma_X\sigma_Y} < p\right) = \frac{Z_v(p)}{v}\sqrt{P_k^X P_k^Y},$$

where $Z_v(p)$ is the confidence level associated with the probability p for a pdf defined by the square root of the product of two χ^2 distributions.

3.2.3. Wavelet coherence (WTC)

Cross wavelet power reveals areas with high common power. Another useful measure is how coherent the cross wavelet transform is in time frequency space. Aguiar- Conraria et al. (2008) defines Wavelet Coherence as “the ratio of the cross-spectrum to the product of the spectrum of each series, and can be thought of as the local (both in time and frequency) correlation between two time-series”.

Following Torrence and Webster (1999) we define the Wavelet Coherence of two time series as

$$R_n^2(s) = \frac{|S(s^{-1}W_n^{XY}(s))|^2}{S(s^{-1}|W_n^X(s)|^2)\cdot S(s^{-1}|W_n^Y(s)|^2)},$$

where S is a smoothing operator. Notice that this definition closely resembles that of a traditional correlation coefficient, and it is useful to think of the wavelet coherence as a localized correlation coefficient in time frequency space. We write the smoothing operator S as

$$S(W) = S_{\text{scale}}(S_{\text{time}}(W_n(s)))$$

where S_{scale} denotes smoothing along the wavelet scale axis and S_{time} denotes smoothing in time. For the Morlet wavelet a suitable smoothing operator is given by

$$S_{\text{time}}(W)|_s = (W_n(s) * c_1^{\frac{-t^2}{2s^2}})|_s,$$
$$S_{\text{scale}}(W)|_n = (W_n(s) * c_2 \Pi(0.6s))|_n,$$

where c_1 and c_2 are normalization constants and Π is the rectangle function. The factor of 0.6 is the empirically determined scale de-correlation length for the Morlet wavelet . In practice both convolutions are done discretely and therefore the normalization coefficients are determined numerically. Since theoretical distributions for wavelet coherency have not been derived yet, to assess the statistical significance of the estimated wavelet coherency, one has to rely on Monte Carlo simulation methods.

However, following Aguiar-Conraria and Soares (2011) we will focus on the Wavelet Coherence, instead of the Wavelet Cross Spectrum. Aguiar-Conraria and Soares (2011) gives two arguments for this: “(1) the wavelet coherency has the advantage of being normalized by the power spectrum of the two time-series, and (2) that the wavelets cross spectrum can show strong peaks even for the realization of independent processes suggesting the possibility of spurious significance tests”.

3.2.4. Cross wavelet phase angle

As we are interested in the phase difference between the components of the two time series we need to estimate the mean and confidence interval of the phase difference. We use the circular mean of the phase over regions with higher than 5% statistical significance that are outside the

COI to quantify the phase relationship. This is a useful and general method for calculating the mean phase. The circular mean of a set of angles $(a_i, i = 1...n)$ is defined as

$$a_m = \arg(X, Y) \text{ with } X = \sum_{i=1}^{n} \cos(a_i) \text{ and } Y = \sum_{i=1}^{n} \sin(a_i).$$

It is difficult to calculate the confidence interval of the mean angle reliably since the phase angles are not independent. The number of angles used in the calculation can be set arbitrarily high simply by increasing the scale resolution. However, it is interesting to know the scatter of angles around the mean. For this we define the circular standard deviation as,

$$s = \sqrt{-2\ln(R/n)},$$

where $R = \sqrt{(X^2 + Y^2)}$. The circular standard deviation is analogous to the linear standard deviation in that it varies from zero to infinity. It gives similar results to the linear standard deviation when the angles are distributed closely around the mean angle. In some cases there might be reasons for calculating the mean phase angle for each scale, and then the phase angle can be quantified as a number of years.

The phase difference between the two series is indicated by arrows. Arrows pointing to the right mean that the variables are in phase. To the right and up, the second variable is leading. To the right and down, the second variable is lagging. Arrows pointing to the left mean that the variables are out of phase. To the left and up, the second variable is lagging. To the left and down, the second variable is leading. In phase indicate that variables will be having cyclical effect on each other and out of phase or anti-phase shows that variable will be having anti-cyclical effect on each other.

3.3. Cointegration Analysis

The two-step Engle Granger procedure searches for parameters α, β, and ρ that yield the best fit to the following model:

$$Y[i] = \alpha + \beta * X[i] + R[i]$$
$$R[i] = \rho * R[i-1] + \epsilon[i]$$
$$\epsilon[i] \sim N(0, \sigma^2)$$

In the first step, alpha and beta are found using a linear fit of X[i] with respect to Y[i]. The residual sequence R[i] is then determined. Then, in the second step, ρ is determined, again using a linear fit. Engle and Granger showed that if X and Y are cointegrated, then this procedure will yield consistent estimates of the parameters. However, there are several ways in which this estimation procedure can fail:

- Either X or Y (or both) may already be mean-reverting. In this case, there is no point in forming the difference Y −βX. If one series is mean-reverting and the other is not,then any non-trivial linear combination will not be mean-reverting.

- The residual series R[i] may not be mean-reverting. In the language of cointegration theory, it is then said to contain a unit root. In this case, there is no benefit to forming the linear combination Y −βX.

- The residual series R[i] may be mean-reverting, but the relation R[i] = ρR[i−1] + ε[i] may not be the right model. In other words, the residual series may not be adequately described by an auto-regressive series of order one. In this case, the parameters α and β will be correct, however the specification for the residuals R[i] will not be.

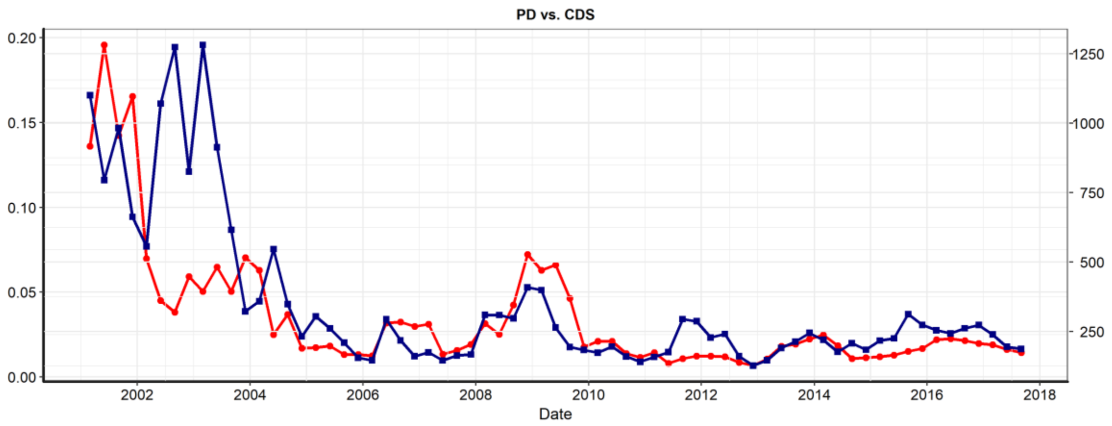
4. Empirical Analysis

For the analysis of the relation between probability at default (PD) of BIST-100 real sector firms and 5-year Turkish credit default swap (CDS), we used quarterly data of the interested variables over the period of January 2001 to September 2017. In total 67 quarters are used. PD is calculated according to the procedure described in section 3.1. Figure 2 indicates calculated PD and CDS series.

- The datas for the calculations of PD's are taken from the Public Disclosure Platform (PDP)

- BIST Stocks Datas & CDS Prices are acquired from Reuters.

Figure 2. PD vs. CDS



Red and blue lines denote BIST Firm PD's and Turkey Sovereign CDS, respectively.

Table 1. shows the correlation and their test analysis results of all the variables. Figure 3. shows us the four-period rolling correlation results.

- When we investigate the PD & CDS results alltogether, historical correlation results are statistically highly significant with %95 confidence level even by parametric or non-parametric models.
- According to the 4 period rolling correlation results, there is a reverse strong relation found between 2002-2006 period on the other hand there is a %90 positive correlation in the 2006-2017 period.
- We can observed a sharp lineer relation between the PD & CDS data's.

Table 1. Correlation Results

	Pearson	Kendall	Spearman
PD vs CDS	0.6333439 [0.0000]	0.5169607 [0.0000]	0.7149413 [0.0000]

Pearson indicates Pearson's product-moment correlation, Kendall and Spearman show Kendall's rank correlation tau and Spearman's rank correlation rho, respectively.

Figure 3. Correlation of PD vs CDS

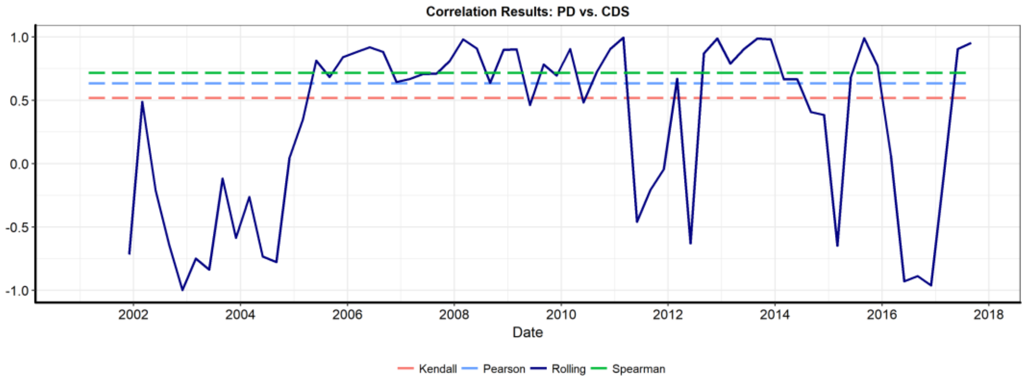


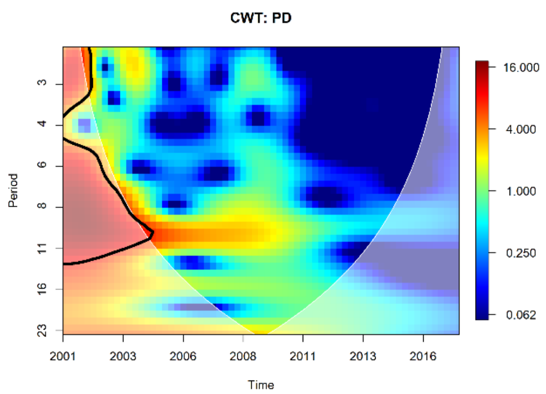
Table *2*. indicates two-step Engle Granger cointegration procedure results.

- When we analyse the results are stationary in the same scales according to the ADF & PP test's.
- According to the lineer regration results model residuals are stationary.
- Because of these results, as a lineer procedure of 2 step Engle-Granger Cointegration Test analysis PD & CDS datas are accepted as cointegrated.
- But cointegration results and in the previous step that we have found with the lineer correlation models show us only there is a lineer relation between PD & CDS data's. Because of that reason in the following step in order to analyse the non-linear dynamic relation with in the time-frequency level we'll research wavelet analysis.

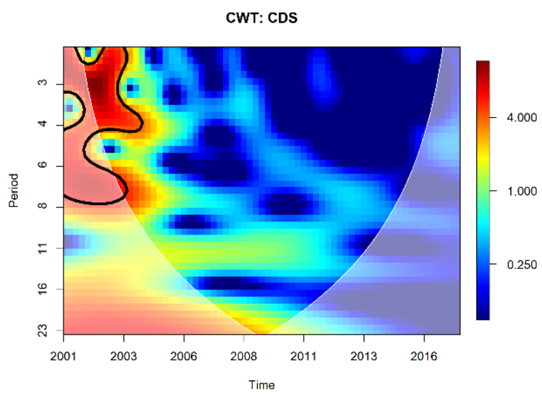
Table 2. Engle Granger Cointegration Results

Parameter	Value	P-Value
$\boldsymbol{\alpha}$	0.0059	0.0128
$\boldsymbol{\beta}$	0.0001	0.0000
$\boldsymbol{\rho}$	0.6597	0.0989
$\boldsymbol{\epsilon_t}$	$N(0,\ 0.023^2)$	

Figure 4. Wavelet Power Spectrum of PD & CDS



(a) WT: PD

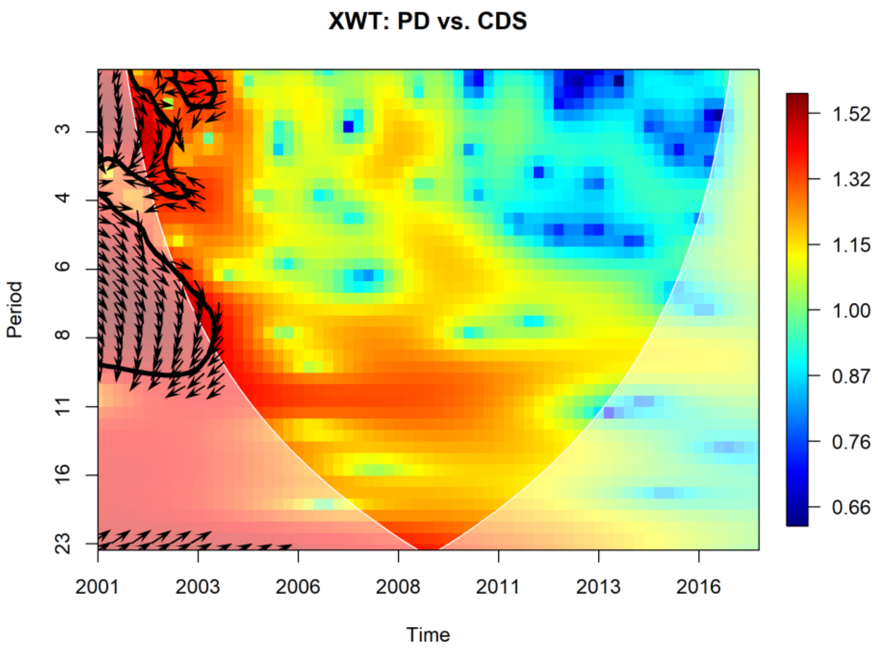


(b) WT: PD

Figure *4* presents the results of continuous wavelet power spectrum of all the series: PD and CDS. The plot of PD (**Figure *4*** (a)) has only small number of significant regions: from 2001 to 2003, the wavelet power spectrum of PD detects high common power in both high and low frequencies for up to 3 years period. Likewise PD, the wavelet power spectrum of CDS (**Figure *4*** (b) are;

- There is only 1 significant region found between the 2001-2003 period. But regarding from PD's energy can only be observed up to 2 years period.
- The significant regions that has been found between the 2001-2003 period shows us the 2001 financial crises makes an energy accumulation both in PD's and CDS data's.

Figure 5.The Cross Wavelet Transform of PD & CDS

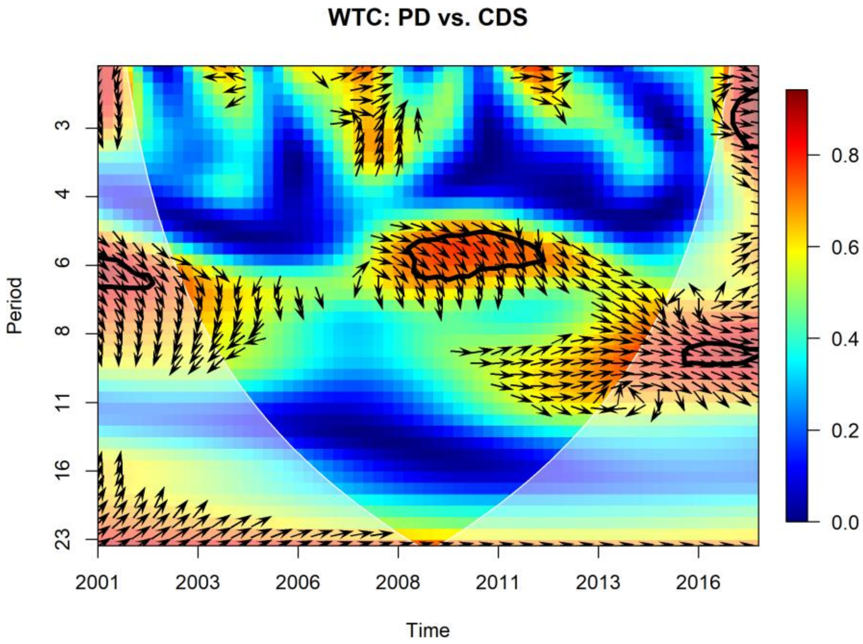


The thick black contour designates the 5% significance level against red noise which is estimated from Monte Carlo simulations using phase randomized surrogate series. The cone of influence, which indicates the region affected by edge effects, is shown with a lighter shade black line. The color code for power ranges from blue (low power) to red (high power).

The results of the cross wavelet transform are given in **Figure *5***.

- As a natural results of wavelet power spectrum cross wavelet transform (XWT), PD & CDS energies shows a high relation between each other up to 2 years in the 2001-2003 period.
- Up to a 1 year high frequency period PD & CDS datas are in negative direction and CDS prices are a leading indicator of PD data's. As a result real sector PD's follow up by a reverse way of CDS price changes.
- Between 1 year and 2 years frequency, PD & CDS data's are on the same movement and the changes in the PD's affects CDS prices changes as well.

Figure 6. The Wavelet Coherence Result of PD & CDS



The thick black contour designates the 5% significance level against red noise which is estimated from Monte Carlo simulations using phase randomized surrogate series. The cone of influence, which indicates the region affected by edge effects, is shown with a lighter shade black line. The color code for power ranges from blue (low power) to red (high power).

Figure ***6***. demonstrates the results of wavelet coherence for PD & CDS. For the pair of PD and CDS and in the significant region marked by thick black contour, about 2001 over 1.5 years cycle the arrows are right-down (the variables are in phase and CDS is lagging); arround 2008-2012 over 1 – 1.5 years period the arrows are right-down indicates that variables are in phase and CDS is lagging. In addition, arround 2016-2017 in low frequency for 2 years cycle the arrows are right-down shows that variables are in phase and CDS is lagging.

- In the 2001 Turkish Banking Crises, 2008 USA Subprime Mortgage Crises and the 2009 European Sovereign Debt Crises Real Sector PD results has the same movement with the CDS prices and PD changes has significant effects with in a defined frequencies on the CDS price changes.
- There is a parallel movement trend between the real sector PD’s and Turkey Sovereign CDS data’s between the 2016-2017 period.

5. Conclusion

In this research we tried to explore the credit default probabilities of Turkish Real Sector firms trading on BIST. We have analyzed the dynamic relations between the calculated prediction data’s & sectoral based ex-post Real Sector Firms default results. Furthermore as it is accepted a credit risk leading indicator, we analyzed the relation between the Real/Banking sector data & Turkish Sovereign CDS values. We used the well-known Merton (1974) model based on the option valuation methodology principles. We also searched the co-integration & dynamic nonlinear wavelet coherency analysis between the firm credit default probability & CDS price changes in the same period.

According to all the analyses that we have found we can summarize the results as it is in the conclusion section;

When we investigate the PD & CDS results all together, historical correlation results are statistically highly significant with %95 confidence level even by parametric or non-parametric models. According to the 4 period rolling correlation results, there is a reverse strong relation found between 2002-2006 period on the other hand there is a %90 positive correlation in the 2006-2017 period. We can observed a sharp lineer relation between the PD & CDS data's.

- When we analyse the results are stationary in the same scales according to the ADF & PP test's.
- According to the lineer regration results model residuals are stationary.
- Because of these results, as a lineer procedure of 2 step Engle-Granger Cointegration Test analysis PD & CDS datas are accepted as cointegrated.

But cointegration results and in the previous step that we have found with the lineer correlation models show us only there is a lineer relation between PD & CDS data's. Because of that reason in the following step in order to analyse the non-linear dynamic relation with in the time-frequency level we researched wavelet analysis.

- There is only 1 significant region found between the 2001-2003 period. But regarding from PD's energy can only be observed up to 2 years period.
- The significant regions that has been found between the 2001-2003 period shows us the 2001 financial crises makes an energy accumulation both in PD's and CDS data's.
- As a natural results of wavelet power spectrum cross wavelet transform (XWT), PD & CDS energies shows a high relation between each other up to 2 years in the 2001-2003 period.
- Up to a 1 year high frequency period PD & CDS data's are in negative direction and CDS prices are a leading indicator of PD data's. As a result real sector PD's follow up by a reverse way of CDS price changes.
- Between 1 year and 2 years frequency, PD & CDS data's are on the same movement and the changes in the PD's affects CDS prices changes as well.
- In the 2001 Turkish Banking Crises, 2008 USA Subprime Mortgage Crises and the 2009 European Sovereign Debt Crises Real Sector PD results has the same movement with the CDS prices and PD changes has significant effects with in a defined frequencies on the CDS price changes.
- There is a parallel movement trend between the real sector PD's and Turkey Sovereign CDS data's between the 2016-2017 period.

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